Reg. No. :

J 3912

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2009.

Second Semester

Civil Engineering

MA 2161 — MATHEMATICS — II

(Common to all branches of B.E./B.Tech.)

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the particular integral of $\left(D^2+2D+1\right)y=e^{-x}\cos x$.

2. Solve the equation $x^2y''-xy'+y=0$.

3. Find the values of a, b, c so that the vector $\overline{F}=(x+y+az)\vec{i}+(bx+2y-z)\vec{j}+(-x+cy+2z)\vec{k}$ may be irrotational.

4. State Green's theorem in a plane.

5. State the Cauchy-Riemann equations in polar coordinates satisfied by an analytic function.

6. Find the invariant points of the transformation $w=\dfrac{2z+6}{z+7}$.

7. Evaluate $\int_C \tan z\, dz$ where C is $|z|=2$.

8. Find the Taylor series for $f(z)=\sin z$ about $z=\dfrac{\pi}{4}$.

9. Find the Laplace transform of $\dfrac{1-\cos t}{t}$.

10. Find the inverse Laplace transform of $\cot^{-1}\left(\dfrac{k}{s}\right)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Solve the equation $(D^2+4)\,y = x^2\cos 2x$. (8)

(ii) Solve the equation $(D^2+a^2)\,y = \tan ax$ by the method of variation of parameters. (8)

Or

(b) (i) Solve the equation $(x^2D^2+3xD+5)\,y = x\cos(\log x)$. (8)

(ii) Solve $\frac{dx}{dt}+y=\sin t$

$$x+\frac{dy}{dt}=\cos t,$$

given that $x=2$ and $y=0$ at $t=0$. (8)

12. (a) (i) Find the angle between the normals to the surface $xy^3z^2=4$ at the points $(-1,-1,2)$ and $(4,1,-1)$. (6)

(ii) Verify Stoke's theorem for $\vec{F}=xy\vec{i}-2yz\vec{j}-zx\vec{k}$ where S is the open surface of the rectangular parallelopiped formed by the planes $x=0$, $x=1$, $y=0$, $y=2$ and $z=3$ above the XY plane. (10)

Or

(b) (i) Find the directional derivative of $\phi=2xy+z^2$ at the point $(1,-1,3)$ in the direction of $\vec{i}+2\vec{j}+2\vec{k}$. (6)

(ii) Verify Gauss divergence theorem for $\vec{F}=x^2\vec{i}+y^2\vec{j}+z^2\vec{k}$ where S is the surface of the cuboid formed by the planes $x=0$, $x=a$, $y=0$, $y=b$, $z=0$ and $z=c$. (10)

13. (a) (i) Find the analytic function $f(z)=P+iQ$, if $P-Q=\frac{\sin 2x}{\cosh 2y-\cos 2x}$. (8)

(ii) Find the bilinear transformation which maps the points $z=0, -i, -1$ into $w=i, 1, 0$ respectively. (8)

Or

(b) (i) If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4\,|f'(z)|^2$. (8)

(ii) Find the image of the half plane $x>c$, when $c>0$ under the transformation $w=\frac{1}{z}$. Show the regions graphically. (8)

14. (a) (i) Evaluate $\int_c \frac{z\,dz}{(z-1)(z-2)^2}$ where c is the circle $|z-2| = \frac{1}{2}$ using Cauchy's integral formula. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{1 - 2x\sin\theta + x^2} (0 < x < 1)$, using contour integration. (8)

Or

(b) (i) Find the Laurent's series of $f(z) = \frac{z^2 - 1}{z^2 + 5z + 6}$ valid in the region $2 < |z| < 3$. (8)

(ii) Evaluate $\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2 + a^2)(x^2 + b^2)}$, using contour integration, where $a > b > 0$. (8)

15. (a) (i) Find the Laplace transform of $te^{-2t} \cos 3t$. (4)

(ii) Find the inverse Laplace transform of $\frac{1}{(s+1)(s^2+4)}$. (4)

(iii) Solve the equation $y'' + 9y = \cos 2t$, $y(0) = 1$ and $y\left(\frac{\pi}{2}\right) = -1$ using Laplace transform. (8)

Or

(b) (i) Find the Laplace transform of

$$f(t) = \begin{cases} t, & \text{in } 0 \le t \le a \\ 2a - t, & \text{in } a \le t \le 2a \end{cases}$$

and $f(t + 2a) = f(t)$. (8)

(ii) Find the Laplace transform of $e^{-4t} \int_0^t t \sin 3t \, dt$. (8)